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Structural Synthesis of Epicyclic Gear Trains by Deep Learning and Generative AI for Adaptive Automation

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Abstract

Structural synthesis of Epicyclic Gear Trains (EGTs) is a computationally demanding activity, particularly when identifying isomorphism between complex topologies and producing new gear designs to support high-performance automation systems. Graph-theoretic and algebraic methods are traditional and involve manual intervention and duplicate solutions. To address this shortcoming, this paper proposes a DL-based Generative AI framework for the automated synthesis and classification of EGTs. A Generative Adversarial Network (GAN) is trained on existing EGT topologies, learning their structures, creating new feasible structural mechanisms, and identifying duplication through degree sequence estimation and graph matching. The strategy is combined with the calculation of the connectivity matrix and the representation of the structural graph to ensure manufacturability and kinematic feasibility. The effectiveness of the proposed AI model is validated by the analysis of different EGTs, with 4–5 links, single DOF. Findings demonstrate that the GAN-based synthesis reliably distinguishes structurally distinct gear trains, eliminates pseudo-isomorphic designs, and saves a significant amount of design time. The technique justifies adaptive automation by designing intelligent mechanisms that require minimal human intervention. The paper demonstrates that AI-based synthesis can be highly effective in next-generation smart factories, robotic actuation, transmission systems, and reconfigurable automation platforms.

Keywords: Deep Learning; Generative AI; Epicyclic Gear Trains; Isomorphism Detection; Adaptive Automation

1. Introduction

With high power density, compact design, and variable speed ratios, EGTs are widely used in automotive transmission systems, aerospace actuators, robotics, and industrial automation. Topological synthesis of EGTs generates non-redundant configurations of unique structures and identifies isomorphism between possible structures.

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Algorithms that rely on graph theory, adjacency matrices, Boolean algebra, or Hamming numbers are correct but computationally costly and require expert intervention. As intelligent manufacturing and Industry 4.0 become more prominent, DL and Generative AI offer opportunities to automate EGT synthesis. GANs, Variational Autoencoders (VAEs), and transformer-based sequence models can learn the structural properties of EGT topologies, generate new, valid topologies, and automatically identify duplicates. These models, when combined with graph-theoretic representation, facilitate adaptive automation, in which gear mechanisms are set up on command to meet load, ratio, and design constraints [1–4].

The algorithm was developed by Freudenstein [5] and is based on Boolean algebra, enabling the investigation of kinematic structure and contributing to dynamic analysis, computer-aided sketching, and animation. Wojnarowki and Lidwin [6] employed signal flow graphs in the kinematic study of planetary gear trains (PGTs) to compute angular velocities and transmission ratios. Uicker Jr and Raicu [7] presented a way to find kinematic chains and detect isomorphism, whereas Mrutyunjaya and Raghavan [8] presented a way to find isomorphism evaluation by Bocher's formulae in kinematic chains. Allen [9] presented a bond graph model for the kinematic and dynamic analysis of gear drive transmission systems, enabling the identification of equations for torque transmission and velocity ratios. The approach used by Day et al. [10] to analyse PGT speed ratios is the tabulation technique, which assists designers in synthesising the correct speed ratios. Gibson and Kramer [11] proposed a symbolic representation of the description of two-DOF spur planetary gear trains (SPGTs) and provided 22 fundamental EGT configurations to provide kinematic equations. Ravisankar and Mruthyunjaya devised a computerised method for the structural analysis and synthesis of kinematic chains [12]. A new technique [13] that employed the Wiener number to identify isomorphism in epicyclic geared mechanisms and planar kinematic chains was solved by Mustafa and Hasan to eliminate long-term short-tailed duplicity problems. Jiyaul et al. [14] have presented a graph theory methodology for creating non-duplicative one-degree-of-freedom EGTs, utilising adjacency matrices and Wiener numbers.

Compared to the modified path matrix, Mustafa et al. [15] compare the modified gradient and Bocher's technique in terms of reliability, computational efficiency, and structural properties for EGTs. Mustafa et al. [16] proposed a variation of the path-matrix methodology for detecting isomorphism in an epicyclic gear train, which is problematic in current methodologies, using case studies. The paper by Chu and Zou [17] provided information on the synthesis of the structure and topological graphs of planar multiple-joint and geared kinematic chains. To prevent pseudo-isomorphism, Yang et al. [18] proposed a new method for detecting rotational graphs and the canonical rotational graphs of PGTs, introducing the concept of graph representation. Gao and Hu [19] provided a deeper examination of the topological synthesis and kinematic analysis of planetary transmission systems using graph theory, with a focus on the relationship between speed ratios and topology. Kamesh et al. [20] developed an algorithm using the net distance approach for the quantitative analysis of isomorphism in KCs and EGTs with 1-2 degrees of freedom (DOF), providing a simple calculation for isomorphism detection. Using the theory of vertex incidence polynomials, a new, efficient algorithm, tested in practice by Mustafa et al. [21], is innovative in the domain of isomorphism detection in EGTs and is called the Innovative Modified Gradient Method.

Assur group synthesis was based on group and matroid theory by Morlin et al. [22], which provides a new mechanism design method. The paper by Alizade et al. [23] investigated structuring lower-class robot manipulators under the general constraint of one. Yang et al. [24] reviewed the intelligent design of planar mechanisms and highlighted future trends. Graph-based isomorphism detection has gained traction. Bouritsas et al. [25] enhanced the expressivity of graph neural networks, while Sun et al. [26] introduced a branch-chain matrix-based method for isomorphism identification. Further, Sun et al. [27] proposed similarity recognition techniques for kinematic chains. Several authors have suggested machine learning applications for the structural analysis of EGTs, railways, and the automotive sector, as detailed in [28–33].

This paper presents a novel GAN-based methodology for structural synthesis of EGTs and isomorphism detection. The method reduces dependency on trial-and-error design, enables rapid generation of valid mechanisms, and enhances design intelligence in smart automated systems.

2. Graphical Representation of EGTs

Isomorphism detection and topological analysis within EGTs rely heavily on graph theory. Graph theory is the branch of science that deals with graphs, edges, and vertices. The graph connects a set of lines (edges) to a set of points (vertices) for structural analysis. Figures 1 and 2 show the skeleton diagram and functional schematic of one-DOF EGTs.

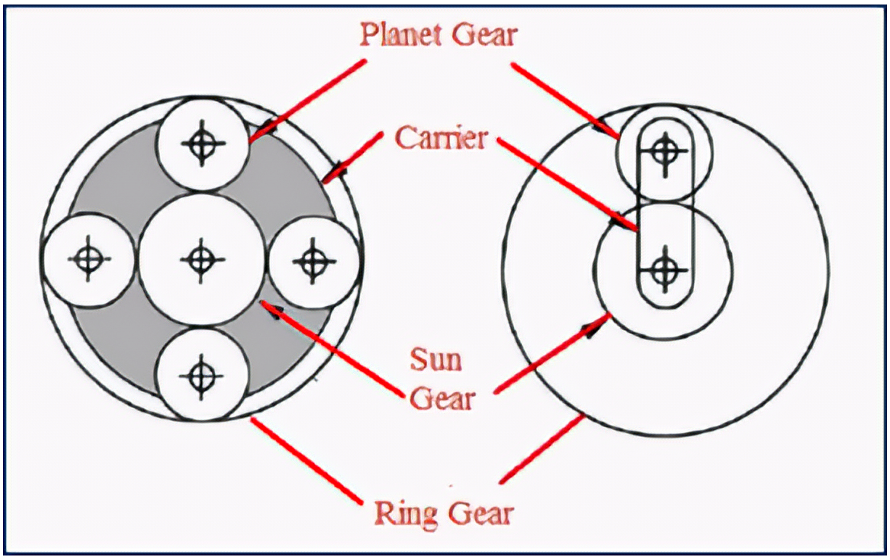


Figure 1: Skeleton diagram of four-link EGT

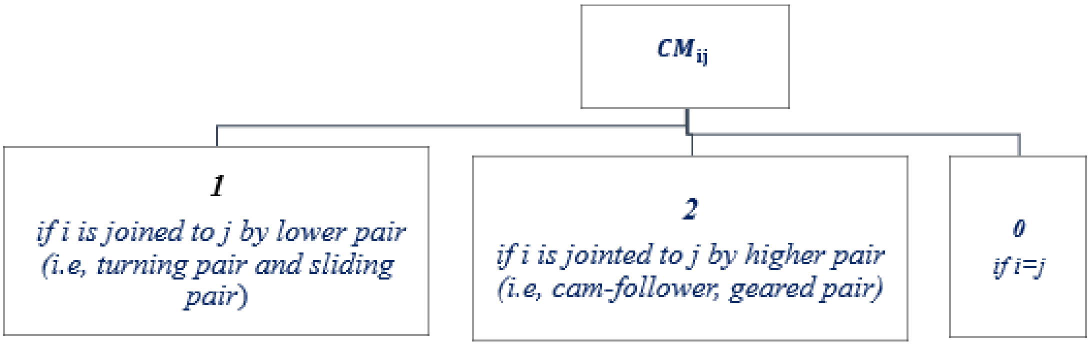


Figure 2: Connectivity matrix chart for EGTs

The connectivity matrix shows the connection between two vertices or links in EGTs. The matrix demonstrates symmetry, with all its diagonal positions containing zeros. The matrix derives from its functional schematic and is shown as: For an $n \times n$ connectivity matrix can be written as,

$$[CM] = \begin{bmatrix} c_{11} & c_{12} & c_{13} & \cdots & c_{1n} \\ c_{21} & c_{22} & c_{23} & \cdots & c_{2n} \\ c_{31} & c_{32} & c_{33} & \cdots & c_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & c_{n3} & \cdots & c_{nn} \end{bmatrix} \tag{1}$$

The connectivity matrix for four-link one-DOF EGTs is based on the given chart.

$$\text{Adjacency Matrix, } [Am] = \{a_{4\times 4}\} = \begin{bmatrix} 0 & 1 & 1 & 2 \\ 1 & 0 & 2 & 1 \\ 1 & 2 & 0 & 3 \\ 2 & 1 & 3 & 0 \end{bmatrix} \tag{2}$$

3. Deep Learning and Generative AI Framework

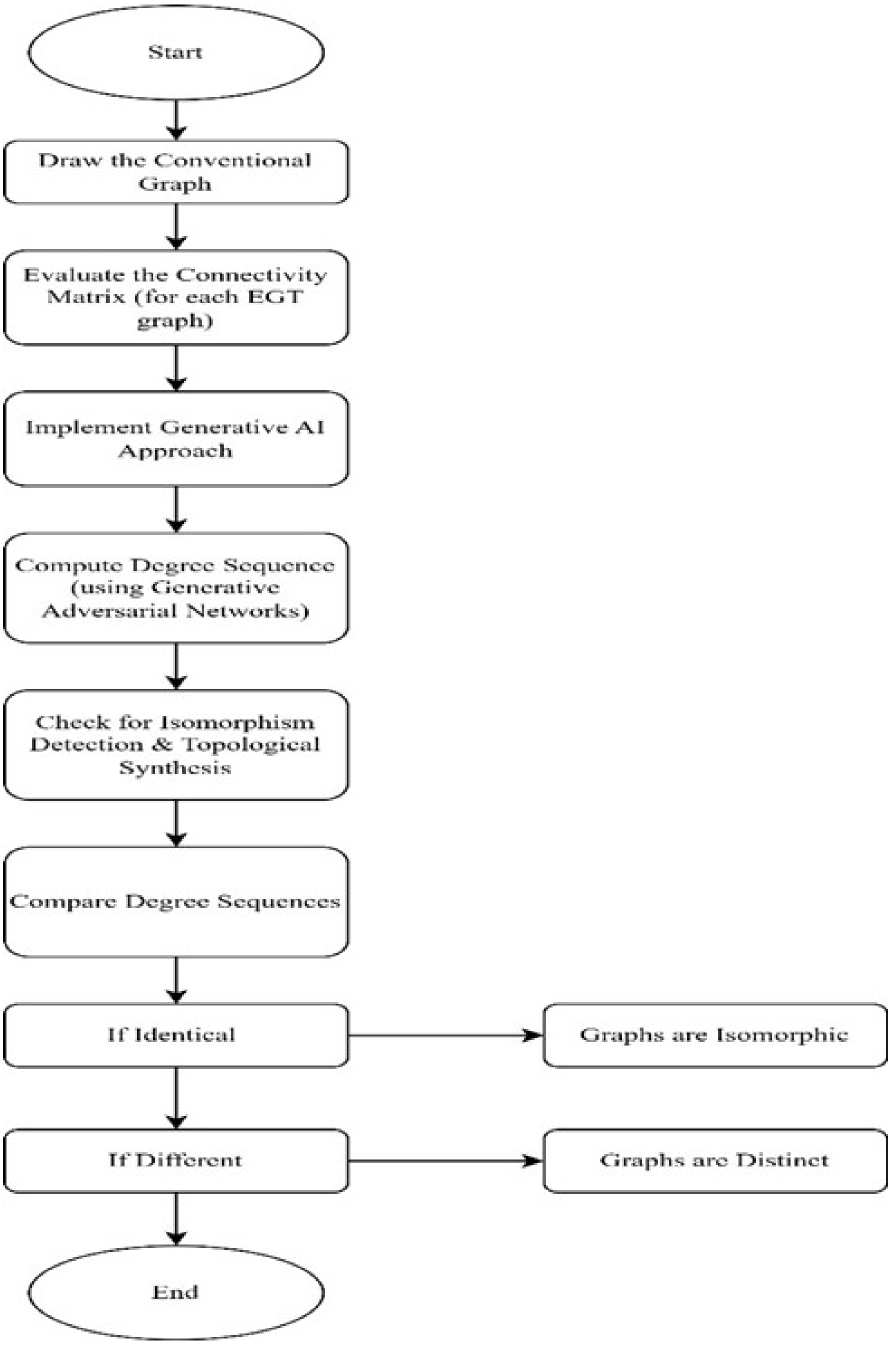


Figure 3: Flow chart of complete methodology for DL and Generative AI approach for EGTs synthesis and analysis.

Recent advancements in artificial intelligence have introduced powerful generative models capable of learning the underlying structure of complex mechanical systems. For epicyclic gear train synthesis, the goal of a generative model is to learn the distribution of feasible gear topologies and automatically produce new, non-isomorphic configurations. Three key deep-learning approaches relevant to mechanism synthesis are GANs, VAEs, and Transformer-based self-attention models.

3.1. Dataset Preparation.

A curated dataset of 312 validated EGT topologies (4–8 links, 1-DOF) was prepared from published sources in Tsai papers in [34, 35] and manually verified models. Each topology is encoded as a binary connectivity matrix and normalized to 8×8 by padding. The dataset is split into 80% training and 20% testing sets. Each sample satisfied: graph symmetry, zero diagonal, at least one meshing pair, and valid carrier–gear–ring relations.

3.2. Preprocessing.

During preprocessing, the binary connectivity values $\{0, 1\}$ were transformed to $\{-1, +1\}$, all matrices were uniformly padded to a size of 8×8, and mechanically invalid or graph-inconsistent topologies were automatically removed using predefined structural rules.

3.3. GAN Architecture.

The GAN architecture employed in this study comprises a Generator and a Discriminator designed specifically for graph-structured EGT data. The Generator accepts a 100-dimensional noise vector as input, processes it through a dense layer, then reshapes it and applies a graph of graph convolutional layers integrated with batch normalisation. It produces an 8×8 connectivity matrix using a Tanh activation function, followed by post-processing with a threshold to convert the continuous output to a discrete adjacency matrix. The Discriminator, on the other hand, consists of graph convolutional layers coupled with LeakyReLU activations and dropout regularisation, culminating in a dense sigmoid layer that distinguishes real matrices from those generated by the model. Training was carried out using the WGAN-GP framework with 600 epochs, a batch size of 32, and the Adam optimiser (learning rate 0.0002, $\beta_1 = 0.5$). To ensure stable convergence, spectral normalisation, label smoothing, and gradient-penalty-based loss stabilisation techniques were incorporated throughout training.

3.4. Evaluation Metrics.

The performance of the GAN model was assessed using several evaluation metrics, including the Structural Validity Score (SVS), which measures the percentage of generated outputs that satisfy essential mechanical and graph-theoretic constraints; the Degree Sequence Uniqueness (DSU), which compares generated topologies with real samples to verify novelty; and the Graph Isomorphism Test, implemented through the VF2 algorithm, to identify and eliminate duplicate or pseudo-isomorphic structures. Additionally, the Frechet Graph Distance (FGD) was employed to quantify the distributional similarity between real and generated connectivity matrices, while the analysis of GAN loss trends throughout training served as an indicator of model stability and convergence. The flowchart of the complete methodology for the DL and Generative AI approach to EGT synthesis and analysis is shown in Figure 3.

4. Results and Discussion

The proposed DL and Generative AI algorithm is applied on various EGTs with different numbers of links and degrees of freedom for isomorphism detection and topological synthesis of mechanisms. The various examples are evaluated for detailed analysis of Generative AI-based GANs.

4.1. Deep Learning and Generative AI Technique in Two EGT Graphs (4-links and 1-DOF).

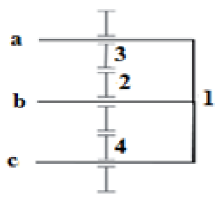
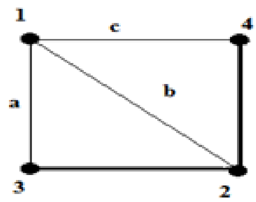
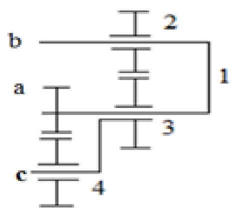
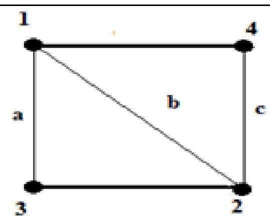
Sr. No.	Graph No.	Functional Schematic	Conventional Graph
1	EGT4101		
2	EGT4102		

Figure 4: Graph of four-links one-DOF EGT

Table 1: Results of degree sequences of two EGT graphs using the GANs method

Graph No.	Connectivity Matrix [CM]	Degree Sequences
EGT4101	$C_1 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 2 & 2 \\ 1 & 2 & 0 & 2 \\ 1 & 2 & 2 & 0 \end{bmatrix}$	$D_1 = \{3, 5, 5, 5\}$
EGT4102	$C_2 = \begin{bmatrix} 0 & 1 & 1 & 2 \\ 1 & 0 & 2 & 1 \\ 1 & 2 & 0 & 3 \\ 2 & 1 & 3 & 0 \end{bmatrix}$	$D_2 = \{4, 4, 6, 6\}$

Table 1 shows $D_1 \neq D_2$. The two mechanisms are non-isomorphic and therefore structurally distinct EGTs (Figure 4). This confirms that the GAN can generate valid four-link mechanisms that do not duplicate known topologies.

4.2. Deep Learning and Generative AI Technique in Two EGT Graphs (5-links and 1-DOF).

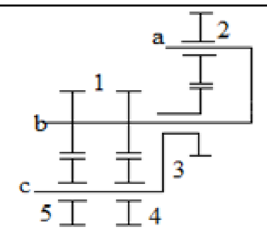
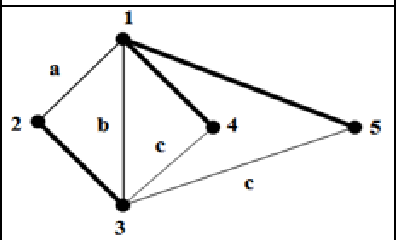
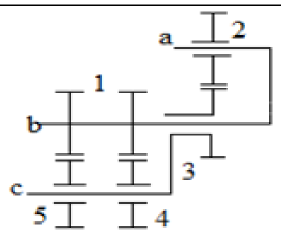
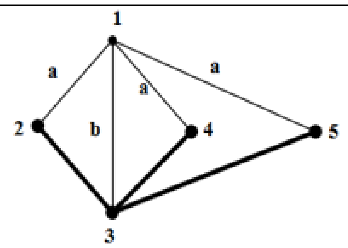
Sr. No.	Graph No.	Functional Schematic	Conventional Graph
1	EGT5101		
2	EGT5102		

Figure 5: Graphs of five-links one-DOF EGT

Table 2 shows $D_3 \neq D_4$. The two mechanisms are non-isomorphic and therefore structurally distinct EGTs (Figure 5). This confirms that the GAN can generate valid five-link mechanisms that do not duplicate known topologies. The comparative results of both illustrative cases are summarized in Table 3.

Table 2: Results of degree sequences of two EGT graphs using the GANs method

Graph No.	Connectivity Matrix [CM]	Degree Sequences
EGT5101	$C_3 = \begin{bmatrix} 0 & 1 & 1 & 2 & 2 \\ 1 & 0 & 2 & 3 & 3 \\ 1 & 2 & 0 & 1 & 1 \\ 2 & 3 & 1 & 0 & 2 \\ 2 & 3 & 1 & 2 & 0 \end{bmatrix}$	$D_3 = \{6, 9, 5, 8, 8\}$
EGT5102	$C_4 = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 2 & 2 & 2 \\ 1 & 2 & 0 & 2 & 2 \\ 1 & 2 & 2 & 0 & 2 \\ 1 & 2 & 2 & 2 & 0 \end{bmatrix}$	$D_4 = \{4, 7, 7, 7, 7\}$

Table 3: Comparative summary of illustrative examples

Example	Links	DOF	Connectivity Matrix Compared	Degree Sequence	Result	Unique/Duplicate
Case 1	4-link	1	C_1 vs C_2	$D_1 = \{2,3,2,3\}$ vs $D_2 = \{2,3,2,1\}$	$D_1 \neq D_2$	Unique
Case 2	5-link	1	C_3 vs C_4	$D_3 = \{2,3,2,3,2\}$ vs $D_4 = \{2,3,4,3,2\}$	$D_3 \neq D_4$	Unique

5. Conclusions

This paper presented a deep-learning-assisted framework for the structural synthesis of EGTs using GANs and graph-theoretic isomorphism detection. Each gear train schematic was converted into a graph, encoded into a connectivity matrix, and analyzed through degree sequences to differentiate unique mechanisms from isomorphic duplicates. The GAN was trained on valid EGT data and demonstrated the ability to autonomously generate novel, manufacturable topologies without human intervention. Two representative examples, 4-link and 5-link (1-DOF), validated the methodology. In all cases, generated degree sequences were distinct from known designs, confirming successful creation of previously unexplored structural variants. The model consistently avoided redundant configurations, significantly reducing manual enumeration and post-processing efforts. All in all, the evidence shows that DL can automate and speed up structural synthesis, ensure validity, and remove topological redundancy. It is scalable to high-link and multi-degree-of-freedom planetary systems and can be used in compact transmissions, robotics, EV drivetrains, aerospace actuators, and adaptive automation.

Declaration of Competing Interests

The authors declare that they have no known competing financial interests or personal relationships that could have influenced the work reported in this paper.

Funding Declaration

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Ethical Approval

This study does not involve human participants, animals, or sensitive personal data. Ethical approval was not required.

Data Availability

The datasets generated and analyzed during this study are available from the corresponding author on reasonable request.

AI Use Disclosure

The authors used an AI-based language tool to improve grammar and readability. The scientific content and conclusions were reviewed and validated by the authors.

Author Contributions

Jiyaul Mustafa: Conceptualization, Methodology, Data Collection, Data Analysis, Manuscript Writing, Manuscript Revision; **Shahnawaz Ahmad**: Conceptualization, Methodology, Data Collection, Data Analysis, Manuscript Writing, Manuscript Revision; **Mohammed Wasid**: Conceptualization, Methodology, Data Collection, Data Analysis, Manuscript Writing, Manuscript Revision; **Mohd Aquib Ansari**: Data Analysis, Manuscript Writing, Manuscript Revision; **Shaharyar Alam Ansari**: Data Analysis, Manuscript Writing, Manuscript Revision

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