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Hybrid Despeckling for Ultrasound Images Using Sticks Filter and Fourth-Order PDE for Enhanced Diagnostic Precision

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Abstract

Speckle noise in ultrasound imaging poses significant challenges by degrading image quality and affecting diagnostic precision. This study evaluates and compares the performance of established despeckling algorithms, including Lee, Kuan, Frost, Non-Local Means, and PMAD filters, as well as advanced techniques such as Fourth-Order Partial Differential Equations (PDEs) and a novel hybrid method combining Sticks filters with Fourth-Order PDE. Quantitative assessment was performed using metrics such as Peak Signal-to-Noise Ratio (PSNR), Mean Squared Error (MSE), Equivalent Number of Looks (ENL), Structural Similarity Index (SSI), Signal-to-Mean Power Index (SMPI), and computational efficiency. Among the evaluated methods, the Lee filter achieved the highest PSNR of 25.05 dB, demonstrating effective noise suppression while preserving the details of the image. The combination of Sticks and Fourth-Order PDE achieved the highest ENL of 0.0331, indicating superior smoothing in homogeneous regions and enhanced contrast. While PMAD exhibited superior speckle suppression with a minimal MSE of 886.49, it introduced slight blurring, compromising structural details. Visual inspections revealed that the hybrid Sticks and Fourth-Order PDE approach delivered exceptional edge preservation and contrast enhancement, outperforming other filters in clinical scenarios such as thyroid nodule analysis. The results demonstrate that the proposed hybrid method addresses critical trade-offs between noise suppression and detail preservation, offering a robust framework to improve the diagnostic utility of ultrasound images. Future research could explore optimizing these algorithms for real-time applications, enabling broader clinical adoption.

Keywords: Speckle Noise Reduction; Ultrasound Image Processing; Partial Differential Equations; Edge Preservation; Thyroid Diagnosis

1. Introduction

Ultrasound imaging is widely recognized as a non-invasive, cost-effective, and radiation-free diagnostic tool, particularly valuable for the detection and evaluation of thyroid disorders [1, 2]. However, the intrinsic presence of speckle noise, which originates from backscattered ultrasound signals, severely degrades image quality, complicating diagnosis and feature extraction [3, 4]. Over the years, numerous despeckling algorithms have been proposed to mitigate this issue. Traditional approaches, such as the Lee filter [5–8] and the Kuan filter [5, 9–12], utilize local statistical measures to reduce noise while attempting to preserve image details. The Frost filter introduces a spatially adaptive kernel to achieve similar objectives, while Non-local means focus on patch-level comparisons to retain finer structural details [13–15]. Several authors [16–19] have used advanced techniques such as anisotropic diffusion and its speckle-specific adaptations to offer selective smoothing mechanisms to preserve edges while minimizing noise. Recent developments in fourth-order partial differential equations (PDEs) have further enhanced despeckling by leveraging higher-order curvature information to retain image fidelity [20–24].

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Despite these advancements, many existing methods suffer from trade-offs between noise suppression, edge preservation, and computational efficiency, limiting their applicability in clinical settings [25–27]. This study addresses these gaps by proposing a hybrid approach combining Sticks filters with Fourth-Order PDE, designed to enhance edge clarity and contrast while minimizing speckle noise. The novelty lies in the synergy of these techniques, which offer superior performance in quantitative metrics and visual inspection. By systematically evaluating the proposed method against established filters, this work provides a robust framework to improve diagnostic precision in ultrasound imaging.

2. Despeckling Filters

Speckle noise in ultrasound images is modeled as multiplicative noise superimposed on the original signal [28]. The observed image $y(x, y)$ is expressed as:

$$y(x, y) = I(x, y) \cdot n(x, y), \tag{1}$$

where $I(x, y)$ is the noise-free image and $n(x, y)$ represents the multiplicative noise. Additive noise, although present, has minimal impact compared to the multiplicative component.

2.1. The Lee Filter

The Lee filter takes advantage of local statistical measures, such as mean and variance, to reduce speckle noise. It uses the Equivalent Number of Looks (ENL) to estimate noise variance and control smoothing [7, 29]. The reconstructed image is calculated as:

$$\hat{I}(x, y) = W(x, y) \cdot I(x, y) + [1 - W(x, y)] \cdot \bar{I}(x, y), \tag{2}$$

where $W(x, y)$ is the weighting function defined as:

$$W(x, y) = 1 - \frac{C_n^2}{C_I^2(x, y)}. \tag{3}$$

Here, $C_I(x, y)$ is the coefficient of variation for the intensity of the image and C_n is the coefficient of variation of noise.

2.2. The Kuan Filter

The Kuan filter transforms the multiplicative noise model into an additive noise model using local statistics. The image reconstruction is similar to the Lee filter [30], but the weighting function is modified as:

$$W(x, y) = \frac{1 - \frac{C_n^2}{C_I^2(x,y)}}{1 + C_n^2}. \tag{4}$$

This modification enhances noise reduction while preserving the details of the image. The least mean square error (LMSE) criterion assesses the filter's effectiveness.

2.3. The Frost Filter

The Frost filter estimates the original image by convolving the noisy image with a spatially adaptive kernel [31]. The reconstructed image is given as:

$$\hat{I}(x, y) = I(x, y) * m(x, y), \tag{5}$$

where $m(x, y)$ is the kernel defined as:

$$m(x, y) = k \cdot \exp\left(-\frac{C_I^2(x_0, y_0)}{(x, y)^2}\right). \tag{6}$$

Here, k is a normalization constant, $C_I(x_0, y_0)$ is the local coefficient of variation, and (x, y) denotes the distance within the kernel window. The filter preserves edges while reducing speckle noise.

2.4. The Non-Local Means Filter

The Non-Local Means (NLM) filter replaces pixel-level comparisons with patch-level comparisons. Calculate the intensity in a pixel as the weighted mean of intensities across the image [32]:

$$\hat{I}(x) = \sum_{j\in\Omega} w(x, x_j) \cdot I(x_j), \tag{7}$$

where $w(x, x_j)$ is the weight assigned to the pixel x_j, based on the similarity between the patches around x and x_j. This approach effectively suppresses speckle noise while retaining fine details.

2.5. Perona-Malik Anisotropic Diffusion

Anisotropic diffusion, proposed by Perona and Malik, smooths the images selectively to preserve the edges [33, 34]. The diffusion process is governed by:

$$\frac{\partial I(x, y; t)}{\partial t} = \nabla \cdot [C(\nabla I(x, y; t)) \cdot \nabla I(x, y; t)], \tag{8}$$

where ∇ and $\nabla\cdot$ denote the gradient and divergence operators, respectively, and $C(\cdot)$ is the diffusion coefficient, defined as:

$$C(x) = \exp\left(-\frac{x}{k}\right), \tag{9}$$

With k as the edge threshold parameter. This method achieves noise reduction while preserving structural details.

2.6. Speckle Reduction Anisotropic Diffusion

Yu and Acton proposed a variation of anisotropic diffusion tailored to ultrasound images to specifically address speckle noise [35]. The diffusion equation is given by:

$$\frac{\partial I(x, y; t)}{\partial t} = \nabla \cdot [q(x, y; t) \cdot \nabla I(x, y; t)], \tag{10}$$

where $q(x, y; t)$ is the instantaneous coefficient of variation, the diffusion rate adapts based on speckle characteristics. This approach reduces speckle noise while maintaining the integrity of the edges and features of the image.

2.7. Fourth-Order Partial Differential Equation

Using fourth-order partial differential equations, a model based on the L_2-curvature gradient. The evolution of the image is governed by:

$$\frac{\partial I(x, y; t)}{\partial t} = -\nabla^4 I(x, y; t) + \nabla \cdot [C(\nabla I(x, y; t)) \cdot \nabla I(x, y; t)], \tag{11}$$

where ∇ represents the biharmonic operator (Laplacian squared), and $C(\cdot)$ is the diffusion coefficient defined as:

$$C(x) = \exp\left(-\frac{\|\nabla I(x, y)\|}{k}\right). \tag{12}$$

Here, k controls the sensitivity to intensity gradients. The fourth-order PDE approach excels in retaining edges while suppressing noise, offering a significant improvement over second-order methods.

2.8. Cascading Uniform Stick Filter and Fourth-Order Partial Differential Equation

The stick filter, proposed by Czerwinski et al. [36], enhances the edge information while reducing the speckle noise. The filter operates by applying a bank of stick-shaped masks in multiple orientations, computing the average intensity for each orientation, and replacing the pixel value with the maximum average. The process is mathematically represented as:

$$g(x, y) = \max_{i=1,\ldots,N} \left[f(x, y) * s_i(x, y)\right], \tag{13}$$

where $f(x, y)$ is the input image, $s_i(x, y)$ represents the i-th stick mask, and $g(x, y)$ is the filtered output. The filtered image is smoothed using a fourth-order partial differential equation for enhanced despeckling and edge preservation. The combined approach minimizes noise while retaining structural details critical for diagnostic accuracy.

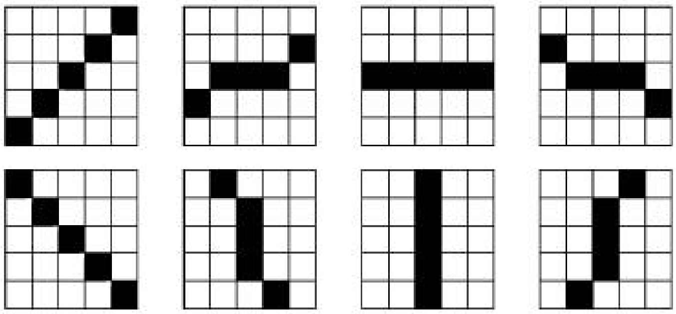

Figure 1: Illustration of stick filter orientations. Each stick mask represents a possible line direction for segmentation.

3. Performance Evaluation Measure

A set of quantitative performance metrics was employed to assess the effectiveness of speckle reduction metrics to evaluate the filter's ability to suppress speckle noise and preserve visual detail and linear structures. While some tests focus on noise reduction, others examine the retention of fine-grained image details. Conflicting results can arise, as a filter that excels in noise suppression may compromise detail preservation. For this study, a speckle-contaminated sample image was used to evaluate filter performance based on the following criteria: Mean, Mean Squared Error (MSE), Peak Signal-to-Noise Ratio (PSNR), Equivalent Number of Looks (ENL), Structural Similarity Index (SSI), Signal-to-Mean Power Index (SMPI), and Elapsed Time (ET). These metrics are defined as $I(x, y)$: Original image and $Y(x, y)$: Denoised image. In addition, a visual quality inspection was conducted to complement the quantitative analysis, ensuring a comprehensive evaluation of the filter's effectiveness.

4. Results and Discussion

Various despeckling techniques' performance was evaluated using synthetic and real-world ultrasound images. The study incorporated a comprehensive assessment based on quantitative metrics, visual inspection, and diagnostic relevance. Quantitative evaluation included metrics such as mean square error (MSE), peak signal-to-noise ratio (PSNR), equivalent number of looks (ENL), structural similarity index (SSI), signal-to-mean power index (SMPI) and elapsed time (ET). Each metric highlights a specific aspect of the filter's effectiveness, ranging from noise suppression to computational efficiency. For example, lower MSE values signify better denoising, while higher PSNR values indicate superior image clarity. ENL measures the smoothness of homogeneous regions and SSI assesses the structural similarity to the original image. The computational cost of the algorithms was captured through elapsed time.

As summarized in Table 1, the quantitative results reveal interesting insights. The Lee filter achieved the highest PSNR, demonstrating its capacity to suppress speckle noise effectively while preserving fine image details. In contrast, the combination of Sticks and Fourth-Order Partial Differential Equations (PDE) produced the highest ENL, reflecting its ability to smooth homogeneous regions while maintaining edge integrity. Fourth-order PDE performed exceptionally well in preserving the mean intensity of the image, underscoring its robustness in retaining image fidelity. While PMAD exhibited superior speckle suppression, it introduced slight blurring, compromising finer structural detail retention. Non-local means achieved high structural similarity scores but at the expense of significant computational overhead, rendering it less feasible for real-time applications. Visual inspection further corroborated the quantitative findings.

Table 1: Performance Metrics for Despeckling Filters

Filter	**Mean**	**MSE**	**PSNR (dB)**	**ENL**	**SSI**	**ET (s)**
Lee	110.72	202.44	25.05	0.0456	0.9038	0.2431
Frost	110.84	408.43	22.04	0.0452	0.9082	11.30
Kuan	110.27	267.41	23.90	0.0473	0.8877	19.87
Non-Local Means	108.85	387.56	22.30	0.0437	0.9235	99.13
PMAD	110.73	886.49	17.07	0.0506	0.8585	0.96
Fourth-Order PDE	110.73	1721.28	16.00	0.0399	1.2296	5.14
Sticks + PDE	128.08	2461.20	14.23	0.0331	1.3499	5.65

The results of applying despeckling filters to synthetic and real-world images are shown in Figures 2 and 3, respectively. The Lee filter provided a balanced trade-off between noise suppression and preservation of structural detail, as evident in Figures 2(c) and 3(b). However, while moderately effective in reducing noise, the Frost filter struggled to retain edge details, particularly in heterogeneous regions. Non-local means demonstrated impressive detail preservation, but their high computational cost limits its practical application. The combination of Sticks and Fourth-Order PDE stood out, delivering exceptional contrast enhancement and edge preservation, making it particularly suitable for clinical scenarios requiring high diagnostic precision.

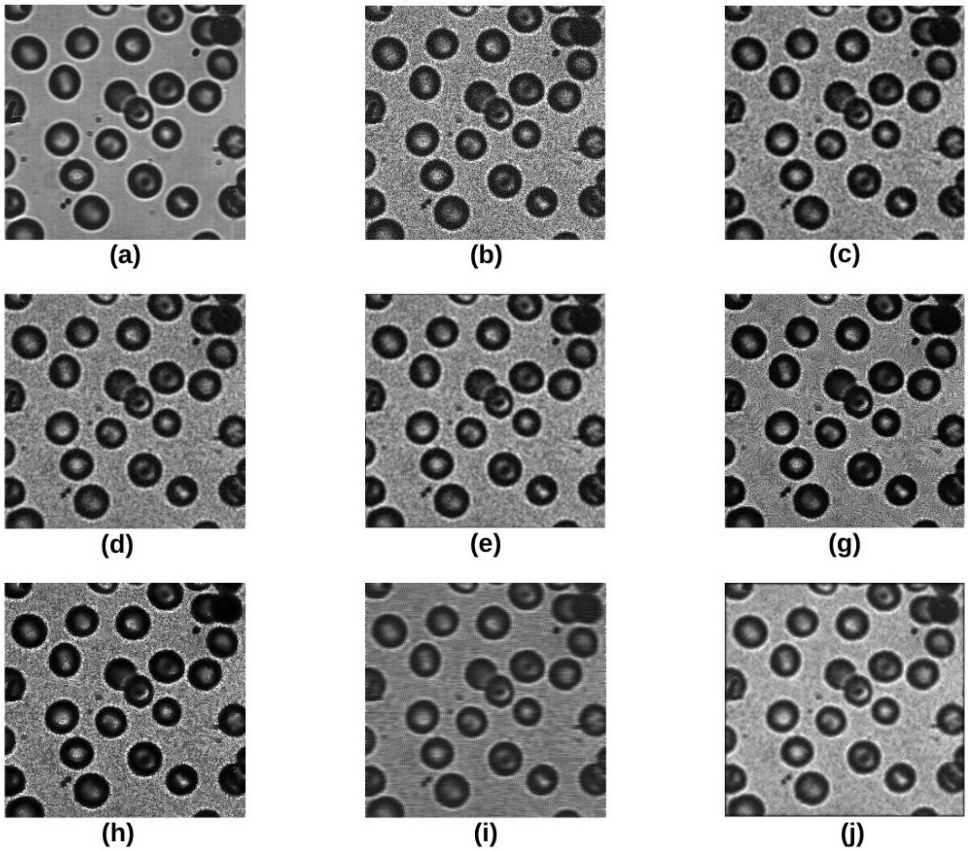

Figure 2: Despeckling results on a blood cell image with 10% speckle noise. (a) Original image, (b) Noisy image, (c) Lee filter, (d) Frost filter, (e) Kuan filter, (f) Non-Local Means filter, (g) PMAD filter, (h) Fourth-Order PDE filter, (i) Sticks combined with Fourth-Order PDE filter.

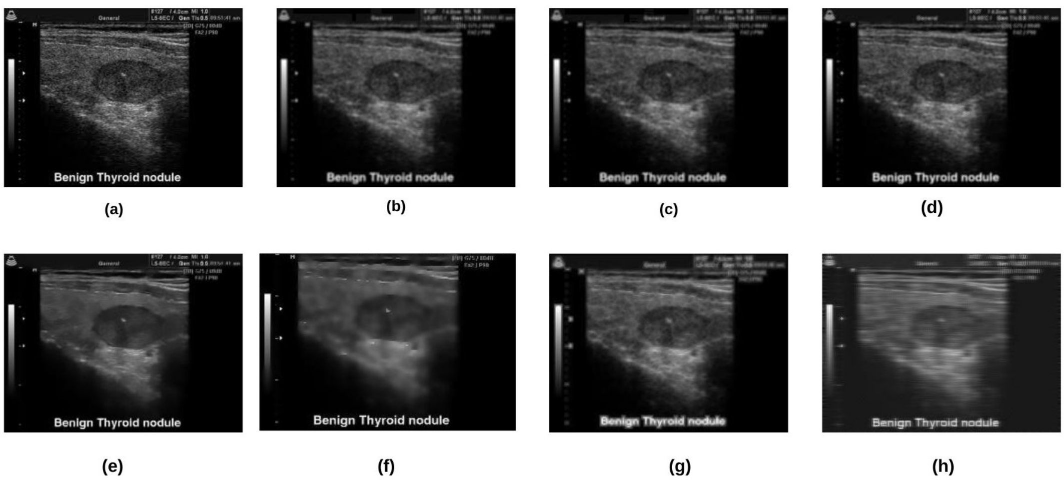

Figure 3: Despeckling results on ultrasound thyroid nodular images. (a) Original image, (b) Lee filter, (c) Frost filter, (d) Kuan filter, (e) Non-Local Means filter, (f) PMAD filter, (g) Fourth-Order PDE filter, (h) Sticks + Fourth-Order PDE filter.

The analysis highlights critical trade-offs between computational efficiency, noise suppression, and structural detail preservation. Filters like PMAD and Fourth-Order PDE, while computationally efficient, require careful parameter tuning to achieve an optimal balance between smoothing and edge retention. However, the combination of Sticks and Fourth-Order PDE demonstrated superior performance in enhancing structural clarity, albeit at a slightly higher computational cost. These trade-offs underscore the need to align the choice of the despeckling algorithm with the application's specific requirements. In the context of clinical relevance, effective reduction in despeckle significantly increases the diagnostic utility of ultrasound images. The findings suggest that while the Lee filter is well suited for general-purpose denoising, Sticks and Fourth-Order PDE offer unparalleled advantages in applications demanding high contrast and precise edge delineation, such as thyroid nodule analysis. This makes the latter an excellent choice for scenarios where accurate diagnosis relies heavily on image quality and structural clarity.

5. Conclusion

This study analyzed several despeckling techniques, combining quantitative metrics and visual evaluation to assess their performance in reducing speckle noise in ultrasound images. Filters such as Lee, Frost, Kuan, Non-Local Means, and PMAD demonstrated their efficacy in suppressing noise. However, this often came at the expense of edge preservation and structural detail retention, particularly in cases of significant noise reduction. Advanced techniques, including Fourth-Order Partial Differential Equations (PDEs) and their combinations, performed better in balancing noise suppression and edge preservation. While Fourth-Order PDEs excel in maintaining image fidelity by preserving edges, their lower PSNR values indicate a trade-off in overall noise reduction performance. Sticks and Fourth-Order PDE emerged as the most effective strategy among the evaluated filters. This method achieved remarkable results in reducing speckle noise, improving edge clarity, and improving image contrast. These attributes make it especially suitable for clinical applications such as thyroid ultrasound imaging, where diagnostic precision is critical. Future work could focus on optimizing the parameters of these algorithms to further improve their computational efficiency and adapt them for clinical use in real time. Additionally, exploring hybrid approaches that integrate the strengths of multiple filters may yield even better results in detail preservation and definition.

Declaration of Competing Interests

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Author Contributions

J. Jai Jaganath Babu: Conceptualization, Supervision, Data Analysis, Writing – Review and Editing; **M. Rohith**: Methodology, Validation, Investigation, Writing – Original Draft; **L. S. Monish Krishnan and L. S. Monish Krishnan**: Software, Visualization, Investigation

References

[1] A. Muhanna, S. S. Ali, A. Khamis, W. A. Suliman, A. Anazi, M. Abdulaziz, A. Qahtani, A. H. Ayedh, A. Muhawwis, L. Waleed, *et al.*, "Exploring the multifaceted applications of ultrasound imaging in medical diagnostics," *International Journal*, vol. 10, 2022.

[2] H. Liu, M. Wang, F. Ji, Y. Jiang, and M. Yang, "Mini review of photoacoustic clinical imaging: a noninvasive tool for disease diagnosis and treatment evaluation," *Journal of Biomedical Optics*, vol. 29, pp. S11522–S11522, 2024.

[3] C. A. Duarte-Salazar, A. E. Castro-Ospina, M. A. Becerra, and E. Delgado-Trejos, "Speckle noise reduction in ultrasound images for improving the metrological evaluation of biomedical applications: an overview," *IEEE Access*, vol. 8, pp. 15983–15999, 2020.

[4] G. Renaud, D. E. Soulioti, and G. Pinton, "Sources of image degradation and their correlation in single-sided ultrasound imaging of heterogeneous tissues," *Innovative Ultrasound Imaging Techniques: Biomedical Applications*, p. 249, 2024.

[5] M. N. Khan and A. Altalbe, "Experimental evaluation of filters used for removing speckle noise and enhancing ultrasound image quality," *Biomedical Signal Processing and Control*, vol. 73, p. 103399, 2022.

[6] S. Banerjee, S. Sinha Chaudhuri, R. Mehra, and A. Misra, "A survey on lee filter and its improved variants," in *Advances in Smart Communication Technology and Information Processing: OPTRONIX 2020*, pp. 371–383, Springer, 2021.

[7] S. V. Parhad, K. K. Warhade, and S. S. Shitole, "Speckle noise reduction in sar images using improved filtering and supervised classification," *Multimedia Tools and Applications*, vol. 83, pp. 54615–54636, 2024.

[8] T. Issam, R. Salah, and M. Brahim, "Filtering techniques to reduce speckle noise and image quality enhancement methods on porous silicon image layers," *Majlesi Journal of Electrical Engineering*, vol. 16, pp. 25–36, 2022.

[9] D. N. K. Hardani, H. A. Nugroho, and I. Ardiyanto, "The impact of filtering for breast ultrasound segmentation using a visual attention model," in *2022 4th International Conference on Biomedical Engineering (IBIOMED)*, pp. 18–23, IEEE, 2022.

[10] L. C. Neves, F. M. Ribas, J. M. Maia, A. J. Zimbico, A. A. Assef, and E. T. Costa, "Enhanced wiener and kuan filters applied with adaptive beamformers for improved contrast and resolution in ultrafast ultrasound images," in *2022 IEEE International Ultrasonics Symposium (IUS)*, pp. 1–4, IEEE, 2022.

[11] P. Paul and B. P. Shan, "Preprocessing techniques with medical ultrasound common carotid artery images," *Soft Computing*, pp. 1–21, 2023.

[12] S. Pradeep and P. Nirmaladevi, "A review on speckle noise reduction techniques in ultrasound medical images based on spatial domain, transform domain and cnn methods," in *IOP Conference Series: Materials Science and Engineering*, vol. 1055, p. 012116, IOP Publishing, 2021.

[13] A. K. Shukla, R. Shree, and J. Narayan, "Combining fusion-based thresholding and non-linear diffusion for improved speckle noise mitigation in sar images," *Applied Sciences*, vol. 14, no. 19, p. 8985, 2024.

[14] J. Arnal and I. Mayzel, "Parallel techniques for speckle noise reduction in medical ultrasound images," *Advances in Engineering Software*, vol. 148, p. 102867, 2020.

[15] H. Choi and J. Jeong, "Despeckling algorithm for removing speckle noise from ultrasound images," *Symmetry*, vol. 12, no. 6, p. 938, 2020.

[16] D. Mittal, V. Kumar, S. C. Saxena, N. Khandelwal, and N. Kalra, "Enhancement of the ultrasound images by modified anisotropic diffusion method," *Medical & Biological Engineering & Computing*, vol. 48, pp. 1281–1291, 2010.

[17] S. Kushwaha and R. K. Singh, "An efficient approach for denoising ultrasound images using anisotropic diffusion and teaching learning based optimization," *Biomedical and Pharmacology Journal*, vol. 10, no. 2, pp. 805–816, 2017.

[18] H. Chen, Y. Wang, J. Shi, J. Xiong, J. Jiang, W. Chang, M. Chen, and Q. Zhang, "Segmentation of lymph nodes in ultrasound images using u-net convolutional neural networks and gabor-based anisotropic diffusion," *Journal of Medical and Biological Engineering*, vol. 41, no. 6, pp. 942–952, 2021.

[19] H. Chen, H. Xu, P. Shi, Y. Gong, Z. Qiu, L. Shi, and Q. Zhang, "3-d gabor-based anisotropic diffusion for speckle noise suppression in dynamic ultrasound images," *Physical and Engineering Sciences in Medicine*, vol. 44, pp. 207–219, 2021.

[20] R. Soorajkumar, P. Krishna Kumar, D. Girish, and J. Rajan, "Fourth order pde based ultrasound despeckling using eni classification," in *2016 International Conference on Signal Processing and Communications (SPCOM)*, pp. 1–5, IEEE, 2016.

[21] P. C. Yadava and S. Srivastava, "Denoising of poisson-corrupted microscopic biopsy images using fourth-order partial differential equation with ant colony optimization," *Biomedical Signal Processing and Control*, vol. 93, p. 106207, 2024.

[22] G. F. Pinton and G. E. Trahey, "A comparison of time-domain solutions for the full-wave equation and the parabolic wave equation for a diagnostic ultrasound transducer," *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, vol. 55, no. 3, pp. 730–733, 2008.

[23] P. C. Yadava, A. Kumar, and S. Srivastava, "Denoising of poisson corrupted micro biopsy image using modified fourth order partial differential equation," in *International Conference on Machine Vision and Augmented Intelligence*, pp. 601–609, Springer Nature Singapore, 2023.

[24] M. Lysaker, A. Lundervold, and X.-C. Tai, "Noise removal using fourth-order partial differential equation with applications to medical magnetic resonance images in space and time," *IEEE Transactions on Image Processing*, vol. 12, no. 12, pp. 1579–1590, 2003.

[25] A. L. Marsden, "Optimization in cardiovascular modeling," *Annual Review of Fluid Mechanics*, vol. 46, no. 1, pp. 519–546, 2014.

[26] R. Zhang, R. A'dawiah, T. W. J. Choo, X. Li, G. Balasundaram, Y. Qi, Y. Goh, R. Bi, and M. Olivo, "Navigating challenges and solutions in quantitative photoacoustic imaging," *Applied Physics Reviews*, vol. 11, no. 3, 2024.

[27] C. Shiranthika, P. Saeedi, and I. V. Bajić, "Decentralized learning in healthcare: a review of emerging techniques," *IEEE Access*, vol. 11, pp. 54188–54209, 2023.

[28] O. V. Michailovich and A. Tannenbaum, "Despeckling of medical ultrasound images," *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, vol. 53, no. 1, pp. 64–78, 2006.

[29] K. Sikhakhane, S. Rimer, M. Gololo, K. Ouahada, and A. M. Abu-Mahfouz, "Evaluation of speckle noise reduction filters and machine learning algorithms for ultrasound images," *IEEE Access*, vol. 12, pp. 81293–81312, 2024.

[30] R. Benes and K. Riha, "Medical image denoising by improved kuan filter," *Advances in Electrical and Electronic Engineering*, vol. 10, no. 1, pp. 43–49, 2012.

[31] A. Kavand and M. Bekrani, "Speckle noise removal in medical ultrasonic image using spatial filters and dncnn," *Multimedia Tools and Applications*, vol. 83, no. 15, pp. 45903–45920, 2024.

[32] D. Dhillon and R. Chouhan, "Edge-preserving image denoising using noise-enhanced patch-based non-local means," *Multimedia Systems*, vol. 29, no. 3, pp. 1025–1041, 2023.

[33] M. J. Black, G. Sapiro, D. H. Marimont, and D. Heeger, "Robust anisotropic diffusion," *IEEE Transactions on Image Processing*, vol. 7, no. 3, pp. 421–432, 1998.

[34] N. Wang, Y. Shang, Y. Chen, M. Yang, Q. Zhang, Y. Liu, and Z. Gui, "A hybrid model for image denoising combining modified isotropic diffusion model and modified perona-malik model," *IEEE Access*, vol. 6, pp. 33568–33582, 2018.

[35] Y. Yu, *Ultrasound image enhancement for detection of contours using speckle-reducing anisotropic diffusion*. PhD thesis, University of Virginia, 2003.

[36] R. N. Czerwinski, D. L. Jones, and W. D. O'Brien, "Ultrasound speckle reduction by directional median filtering," in *Proceedings, International Conference on Image Processing*, vol. 1, pp. 358–361, IEEE, 1995.